Nanyang Technological University

SPMS/Division of Mathematical Sciences

2015/16 Semester 1 **MH1810 Mathematics I** **Tutorial 6**

---

1. Determine the following limits at infinity.
   (a) $\lim\limits_{x\to\infty} \dfrac{3x+5}{x-4}$
   (b) $\lim\limits_{x\to\infty} \dfrac{x^3-2x+3}{5-2x^2}$
   (c) $\lim\limits_{x\to\infty} \dfrac{x+2}{\sqrt{9x^2+1}}$
   (d) $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+4}}$
   (e) $\lim\limits_{x\to\infty} \left(\sqrt{x^4+6x^2}-x^2\right)$
   (f) $\lim\limits_{x\to\infty} \dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$
   (g) $\lim\limits_{x\to\infty} [\ln(2+x)-\ln(1+x)]$
2. Find the domain of $g(x)=\dfrac{\sqrt{x+3}}{x^2-3x-10}$.
3. Use the Intermediate Value Theorem to show that there is a root of the equation $(1-x)^3=\sin x$ in the interval $(0,1)$.
4. Explain why the equation $x^3-15x+1=0$ has three solutions in the interval $[-4,4]$.
5. Use the Intermediate Value Theorem to show that the two graphs $y=e^x$ and $y=-x$ intersect.
6. Suppose that a function f is continuous on the closed interval $[0,1]$ and $0\le f(x)\le 1$ for every $x\in[0,1]$. Is it true that $f(c)=c$ for some $c\in[0,1]$? Justify your answer.
7. (a) Write down the definition of $f'(0)$ and use it to find $f'(0)$ where $f(x)=1-x^3$.
   (b) Use the value $f'(0)$ you have found in part (a) to write down an equation of the tangent line to the curve $y=1-x^3$ at the point $(0,1)$.
8. Use the *definition of derivative* to find the first derivative of the following functions:
   (a) $f(x)=\dfrac{1}{5-3x}$
   (b) $g(x)=\sqrt{x^2+3}$.
9. Consider the function f where $f(x)=x|x|$. Is f differentiable at $x=0$? If it is, determine $f'(0)$.
10. Consider f which is defined as follows:

$$f(x)=\begin{cases} \dfrac{e^x}{2+x} & \text{if } x\ge 0,\\ \cos(1-e^{\pi x}) & \text{if } x<0. \end{cases}$$

Is f differentiable at $x=0$?

Answers

1. (a) 3
   (b) $-\infty$
   (c) $\frac{1}{3}$
   (d) -1
   (e) 3
   (f) 1
   (g) 0

   .....

2. The domain is $[-3, \infty) \backslash \{-2, 5\}$.

   .....

6. TRUE

   .....

7. (a) $f'(0) = 0$
   (b) $y = 1$

   .....

8. (a) $\frac{3}{(5-3x)^2}$
   (b) $\frac{x}{\sqrt{x^2+3}}$

   .....

9. f is differentiable at $x = 0$ and $f'(x) = 0$

   .....

10. Not diffrentiable at $x = 0$.