Nanyang Technological University

SPMS/Division of Mathematical Sciences

2015/16 Semester 1 **MH1810 Mathematics I** **Tutorial 10**

Please be remined that there will be a 15-minute quiz during the tutorial session.

Topics Fundamental Theorem of Calculus, Basic integration techniques: Substitution, by-parts, partial fractions.

PRACTICE MAKES PERFECT!! To master techniques in integration, firstly, we have to familiarize with the antiderivatives of basic functions. This is important in the technique by substitution and integration-by-parts. Secondly, understand various basic techniques in integration. Through practice and observation, you will be able to summarize when to use which techniques. Sometimes, we have to try out different techniques.

1. Find each of the following derivatives.

   (a) $\dfrac{d}{dx}\left(\displaystyle\int_1^x (2+t^4)^5\, dt\right)$

   (b) $\dfrac{d}{dx}\left(\displaystyle\int_{1/x^2}^0 \sin^3 t\, dt\right)$

   (c) $\dfrac{d}{dx}\left(\displaystyle\int_{\cos x}^{5x} \cos(u^2)\, du\right)$

2. Find a function $f(x)$ and a value for the constant c such that

$$\int_c^x tf(t)\ dt = \sin x - x\cos x - \frac{1}{2}x^2, \text{ for all real } x.$$

3. Evaluate the following definite integrals.

   (a) $\displaystyle\int_1^3 \left(5 - \frac{x}{2} + \frac{3}{x^2} - \frac{1}{x}\right)\ dx$

   (b) $\displaystyle\int_{-1}^1 \left(x^2 - 2x + 3\right)\ dx$

   (c) $\displaystyle\int_1^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\ dx$

   (d) $\displaystyle\int_0^{\pi/4} \left(1 + \cos x - \tan^2 x\right)\ dx$

   (e) $\displaystyle\int_0^{\pi/4} (\sin(2x) - \cos(5x))\ dx$

   (f) $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin 2x}{2\sin x} dx$

   (g) $\displaystyle\int_{-1}^0 (2^u + e^u)\ du$

   (h) $\displaystyle\int_{-\pi}^{\pi/2} f(x)\, dx$ where $f(x) = \begin{cases} e^x & \text{if } -\pi \le x \le 0, \\ \cos x & \text{if } 0 < x \le \pi. \end{cases}$

(i) $\displaystyle\int_0^{0.5} \frac{1}{\sqrt{1-x^2}}\ dx$

(j) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{1+x^2}\ dx$

4. Evaluate each of the following integrals by an appropriate substitution.

(a) $\displaystyle\int \frac{\tan^{-1} x}{1+x^2}\ dx$

(b) $\displaystyle\int \frac{e^x}{e^x+1}\ dx$

(c) $\displaystyle\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\ dx$

(d) $\displaystyle\int\ 7\sqrt{7x-1}\ dx$

(e) $\displaystyle\int \frac{4x^3}{(x^4+1)^2}\ dx$

(f) $\displaystyle\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}} dx$

5. Evaluate the following integrals.

(a) $\displaystyle\int\ x \ln x\ dx$

(b) $\displaystyle\int \sin^{-1} y\ dy$

(c) $\displaystyle\int\ xe^{-x}\ dx$

(d) $\displaystyle\int\ x\sin(\pi x)\, dx$

(e) $\displaystyle\int \sin(\ln x)\ dx.$

6. Evaluate the integrals.

(a) $\displaystyle\int\ \frac{x-1}{x^2+3x+2} dx$

(b) $\displaystyle\int \frac{x+4}{x^2+5x+6}\ dx$

(c) $\displaystyle\int \frac{x^2}{(x-3)(x+2)^2} dx$

(d) $\displaystyle\int \frac{x^3}{(x+1)^3} dx$

(e) $\displaystyle\int \frac{1}{x^2+16}\ dx$

(f) $\displaystyle\int \frac{1}{x^2+2x+5}\ dx$

(g) $\displaystyle\int\ \frac{x}{x^2+4x+13} dx$

7. Evaluate $\int x^3 \tan^{-1} x \; dx$.

Answers

1. (a) $(2+x^4)^5$
   (b) $\frac{2}{x^3}\sin^3(\frac{1}{x^2})$
   (c) $5\cos 25x^2 + (\sin x)\cos(\cos^2 x)$

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2. $f(x) = \sin x - 1, c = 0$

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3. (a) $10 - \ln 3$
   (b) $\frac{20}{3}$
   (c) $\frac{64}{3}$
   (d) $\frac{\pi}{2} + \frac{1}{\sqrt{2}} - 1$
   (e) $\frac{1}{2} + \frac{1}{5\sqrt{2}}$
   (f) -1
   (g) $\frac{1}{2\ln 2} + 1 - \frac{1}{e}$
   (h) $2 - e^{-\pi}$
   (i) $\frac{\pi}{6}$.
   (j) $\frac{\pi}{12}$.

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4. (a) $\frac{(\tan^{-1} x)^2}{2} + C$
   (b) $\ln(1+e^x) + C$
   (c) $\frac{\pi^2}{72}$
   (d) $\frac{2}{3}(7x-1)^{3/2} + C$
   (e) $\frac{-1}{x^4+1} + C$
   (f) $\frac{3(1+\sqrt{x})^{4/3}}{2} + C$

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5. (a) $\frac{x^2\ln x}{2} - \frac{x^2}{4} + C$
   (b) $\int \sin^{-1} y \; dy$
   Integration by parts:
   $y\sin^{-1} y + \sqrt{1-y^2} + C$
   (c) $-xe^{-x} - \int (-e^{-x})dx = -xe^{-x} - e^{-x} + C$
   (d) $\frac{-x\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C$
   (e) $\frac{1}{2}(x\sin(\ln x) - x\cos(\ln x)) + C$

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6. (a) $-2\ln|x+1| + 3\ln|x+2| + C$
   (b) $-\ln|x+3| + 2\ln|x+2| + C$

(c) $\frac{9}{25}\ln|x-3|+\frac{16}{25}\ln|x+2|+\frac{4}{5(x+2)}+C$

(d) $x-3\ln|x+1|-\frac{3}{x+1}+\frac{1}{2(x+1)^2}+C$

(e) $\frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right)+C$

(f) $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C$

(g) $\frac{1}{2}\ln(x^2+4x+13)-\frac{2}{3}\tan^{-1}(\frac{x+2}{3})+C$

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7. $\frac{1}{4}\Big((x^4-1)\tan^{-1}x-(\frac{x^3}{3}-x)\Big)+C$