BBMerlion

Math A Level

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QUIZ 2

Vectors

Complex Numbers

Differentiation

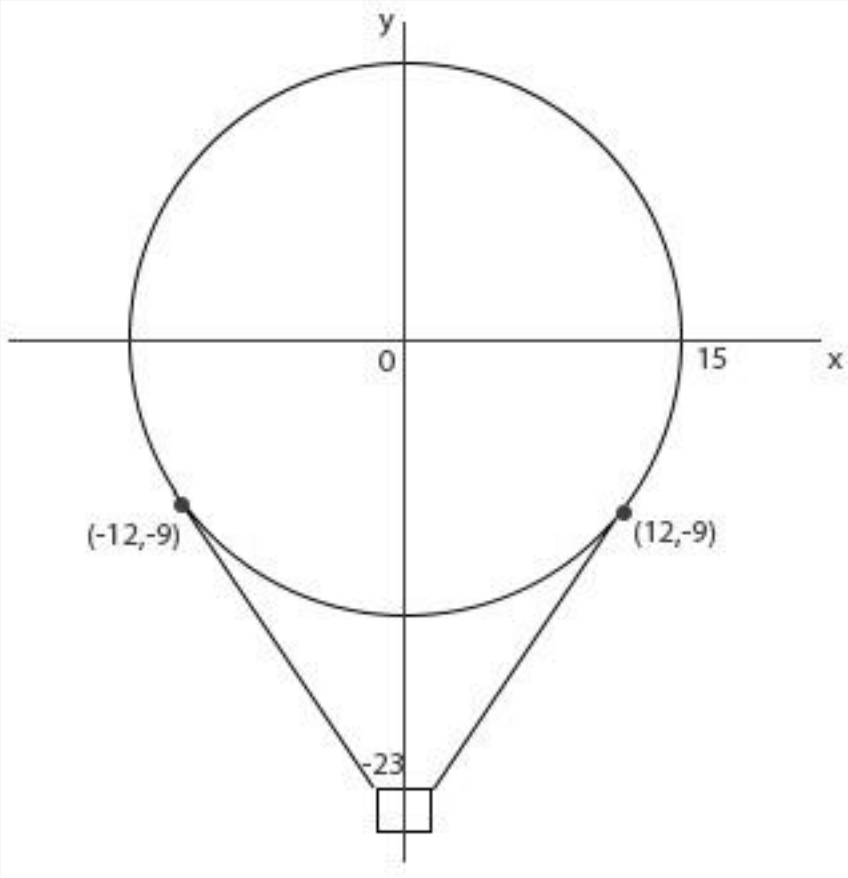
Question 1

a. Show that

$$-\frac{1}{2} \le \frac{x}{1+x^2} \le \frac{1}{2}$$

for every value of x.

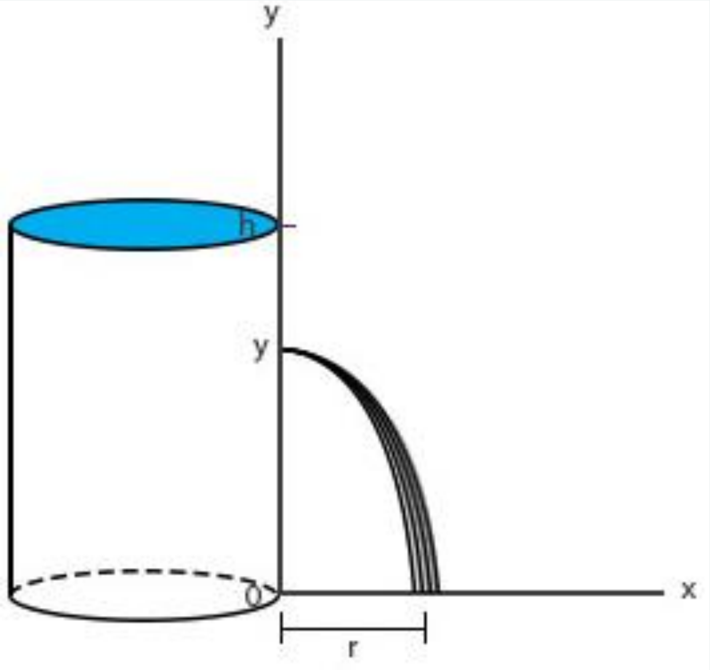
Question 1



Question 1

- What is the width of the box below the circle balloon? The cables connecting the box and balloon are tangent to the balloon.

Question 1



Question 1

C. In what value of y that the water gushing from the tank will hit the ground as far as possible? (Hint: how long does it take for water to hit ground after flowing out of tank?)

The exit velocity of the water is $\sqrt{64(h-y)}$

Question 2

a. If an external force F acts upon a system whose mass varies with time, Newton's law of motion is

$$\frac{d(mv)}{dt} = F + (v + u)\frac{dm}{dt}$$

In this equation, m is the mass of the system at time t, v is its velocity, and v + u is the velocity of the mass that is entering (or leaving) the system at the rate dm/dt. Suppose that a rocket of initial mass m_0 mass starts from rest, but is driven upward by firing some of its mass directly backward at the constant rate of dm/dt = -b units per second and at constant speed relative to the rocket u = -c. The only external force acting on the rocket is F = -mg due to gravity. Under these assumptions, show that the height of the rocket above the ground at the end of t seconds (t is small compared to m_0/b) is

$$y = c\left[t + \frac{m_0 - bt}{b}\ln\frac{m_0 - bt}{m_0}\right] - \frac{1}{2}gt^2$$

Question 2

b. The result of the movement of a dissolved substance across a cell's membrane is described by the equation

$$\frac{dy}{dt} = k\frac{A}{V}(c - y)$$

where y is the changing variable and k, A, V, and c are all constants. Solve the equation for y(t), using y_0 to denote y(0) and find the steady-state concentration, $\lim_{t \longrightarrow \infty} y(t)$.

Question 3

a. Show that the distance between the parallel planes $Ax + By + Cz = D_1$ and $Ax + By + Cz = D_2$ is

$$d = \frac{|D_1 - D_2|}{|A\boldsymbol{i} + B\boldsymbol{j} + C\boldsymbol{k}|}$$

b. Find the distance between the planes $2x + 3y - z = 6$ and $2x + 3y - z = 12$.

c. Find an equation for the plane parallel to the plane $2x - y + 2z = -4$ if the point (3, 2, -1) is equidistant from the two planes.

d. Write equations for the planes that lie parallel to and 5 units away from the plane $x - 2y + z = 3$

Question 4

a. Solve the equation

$$(1-\sqrt{3}i)z^3 = -4+4i$$

Express z as $r\,(\cos q\pi + i\sin q\pi)$, where $r > 0$, q is a rational number and $q \in (-1, 1]$

b. Let $\alpha = 1 + \sqrt{3}i$ and $\beta = \frac{1}{2} - \frac{i}{2}$.

(i.) Express the complex numbers α and β in trigonometric form, $r\,(\cos\theta + i\sin\theta)$ where $r > 0$ and $\theta \in (-\pi, \pi]$

(ii) Simplify $\alpha^3\beta^4$, in the form $x + iy$.

(iii) Find a non-zero polynomial $P(z)$ with real coefficients such that

$$P(\alpha) = P(\beta) = 0$$

References

Thomas Calculus Early Transcedentals 12th Edition