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Math A Level

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QUIZ 1 SOLUTION

Partial Fraction

Function and Graph

Sequence and Series

Question 1

a. $\frac{2x+5}{(x-2)(x+1)} = \frac{A_1}{x-2} + \frac{A_2}{x+1}$

$$= \frac{A_1(x+1) + A_2(x-2)}{(x-2)(x+1)}$$

we take the numerator:

$$2x + 5 = A_1x + A_1 + A_2x - 2A_2 = (A_1 + A_2)x + A_1 - 2A_2$$

$$2 = A_1 + A_2$$

$$5 = A_1 - 2A_2$$

$$A_1 = 3, A_2 = -1$$

Therefore, $\frac{2x+5}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{-1}{x+1}$

Question 1

b. $\frac{5x^2+17x+15}{(x+2)^2(x+1)} = \frac{A_1}{(x+2)} + \frac{A_2}{(x+2)^2} + \frac{A_3}{(x+1)}$

$$= \frac{A_1(x+2)(x+1) + A_2(x+1) + A_3(x+2)^2}{(x+2)^2(x+1)}$$

we take the numerator:

$$5x^2 + 17x + 15 = A_1\left(x^2 + 3x + 2\right) + A_2(x+1) + A_3(x^2 + 4x + 4)$$

$$5 = A_1 + A_3$$

$$17 = 3A_1 + A_2 + 4A_3$$

$$15 = 2A_1 + A_2 + 4A_3$$

$$A_1 = 2, A_2 = -1, A_3 = 3$$

Therefore, $\frac{5x^2+17x+15}{(x+2)^2(x+1)} = \frac{2}{(x+2)} + \frac{-1}{(x+2)^2} + \frac{3}{(x+1)}$

Question 1

c. $\frac{x}{(x^2-x+1)(3x-2)} = \frac{A_1x+A_2}{x^2-x+1} + \frac{A_3}{3x-2}$

$$= \frac{(A_1x + A_2)(3x - 2) + A_3(x^2 - x + 1)}{(x^2 - x + 1)(3x - 2)}$$

we take the numerator:

$$x = 3A_1x^2 - 2A_1x + 3A_2x - 2A_2 + A_3\left(x^2 - x + 1\right)$$

$$0 = 3A_1 + A_3$$

$$1 = -2A_1 + 3A_2 - A_3$$

$$0 = -2A_2 + A_3$$

$$A_1 = -\frac{2}{7}, A_2 = \frac{3}{7}, A_3 = \frac{6}{7}$$

Therefore, $\frac{x}{(x^2-x+1)(3x-2)} = \frac{-\frac{2}{7}x+\frac{3}{7}}{x^2-x+1} + \frac{\frac{6}{7}}{3x-2}$

Question 1

d. Using long division,

$$\frac{2x^4+3x^2+1}{x^2+3x+2} = 2x^2 - 6x + 17 - \frac{39x+33}{x^2+3x+2}$$

Using partial fraction method,

$$\frac{2x^4+3x^2+1}{x^2+3x+2} = 2x^2 - 6x + 17 - \frac{45}{x+2} + \frac{6}{x+1}$$

Question 2

a. $(f \circ g)(x) = \frac{1-4x(1-x)}{1+4x(1-x)} = \frac{4x^2-4x+1}{-4x^2+4x+1}$

$$(g \circ f)(x) = 4\left(\frac{1-x}{1+x}\right)\left(1-\left(\frac{1-x}{1+x}\right)\right)$$

b. $g(x) = x + 2$

Question 2

c. $f(x) = -\frac{9x-3}{7x+6} = u$

$$-9x + 3 = 7xu + 6u$$

$$-9x - 7xu = 6u - 3$$

$$x = -\frac{6u-3}{7u+9}, \qquad f^{-1}(x) = -\frac{6x-3}{7x+9}$$

$$g(x) = 3x^5 - 9 = u$$

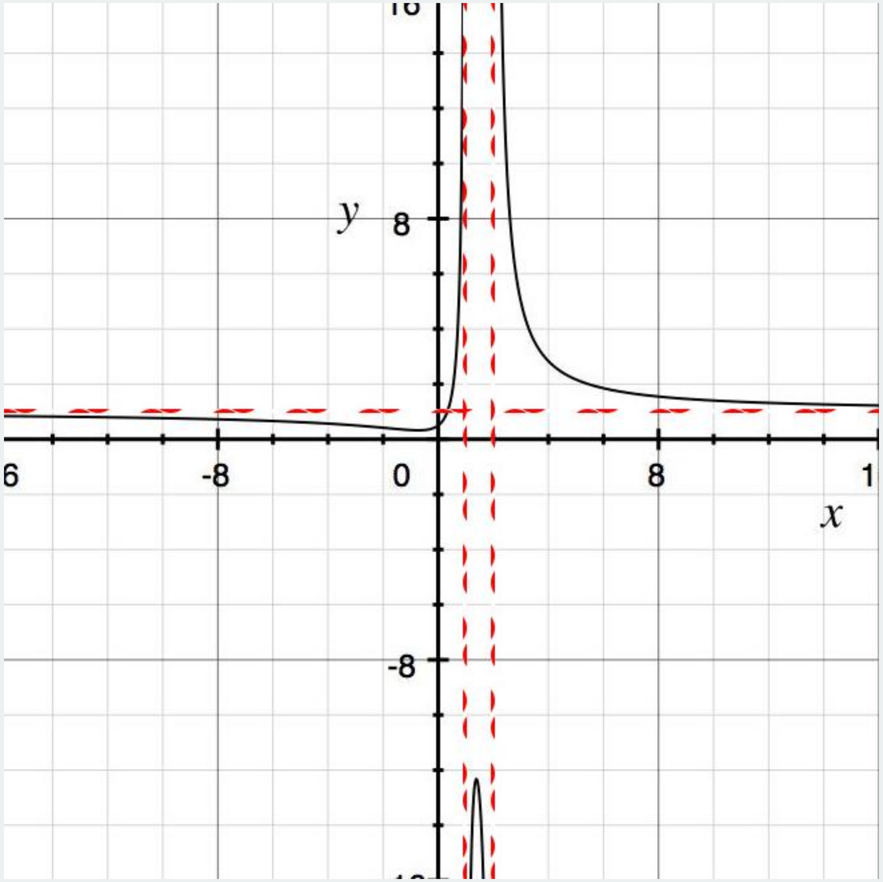
$$x = \sqrt[5]{\frac{u+9}{3}}, \qquad g^{-1}(x) = \sqrt[5]{\frac{x+9}{3}}$$

$$h(x) = \sqrt[3]{9x-7} = u$$

$$9x - 7 = u^3$$

$$x = \frac{u^3+7}{9}, \qquad h^{-1}(x) = \frac{u^3+7}{9}$$

Question 3



Question 3

a. Vertical asymptote: x=1 and x=2

Horizontal asymptote: y=1

Turning point:

$$\frac{dy}{dx} = \frac{2x(x-1)(x-2) - (2x-3)(x^2+1)}{f(x)}$$

$$= 0$$

$$-3x^2 + 2x + 3 = 0$$

$$x_1 = -0.72 \; and \; x_2 = 1.39$$

Question 3

b. At (1, 1),

$$1 = 1 + a + b, \qquad a + b = 0$$

We find the gradient by deriving the function.

$$1 = 2y\frac{dy}{dx} + a\frac{dy}{dx}, \qquad \frac{dy}{dx} = \frac{1}{2y + a}\ (parabola\ 1)$$

$$\frac{dy}{dx} = 2x\ (parabola\ 2)$$

$$\frac{2x}{2y + a}\ at\ (1, 1) = \frac{2}{2 + a} = -1$$

$$a = -4, \qquad b = 4$$

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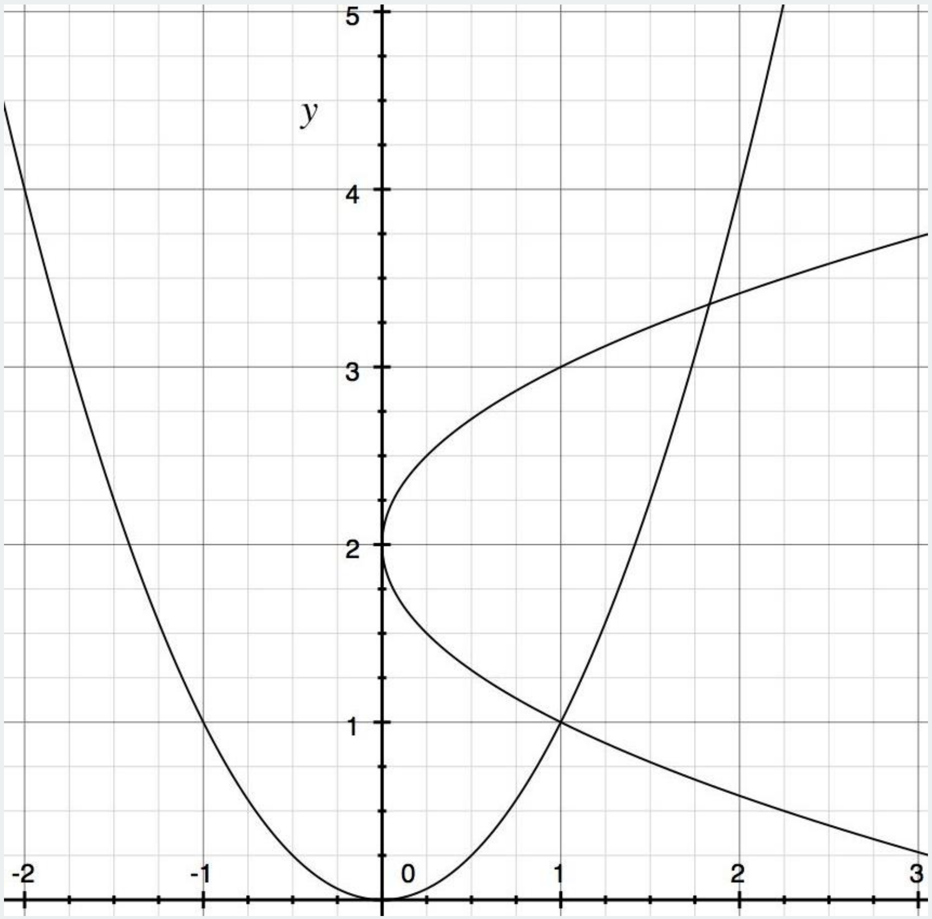
Question 3



Question 4

a. $U_6 = ar^5 = 16\left(-\frac{1}{2}\right)^5 = -\frac{1}{2}$

$$U_7 = ar^6 = \frac{1}{4}$$

$$U_8 - U_{10} = -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}$$

The sequence is convergent because the value will converge to 1 value

Question 4

b. $\sum_{r=10}^{17} r^3$

$$\sum_{r=10}^{17} r^3 = \sum_{r=1}^{17} r^3 - \sum_{r=1}^{9} r^3$$

$$= \frac{1}{4}(17)^2(17+1)^2 - \frac{1}{4}(9)^2(9+1)^2 = 21384$$

Question 4

c. Let P_n be $\left(R(\cos t + i\sin t)\right)^n = R^n(\cos nt + i\sin nt)$
When n = 1,

$$R(\cos t + i\sin t) = R(\cos t + i\sin t)$$

Assume P_k is true.
For P_{k+1},

$$LHS = \left(R(\cos t + i\sin t)\right)^{k+1} = R(\cos t + i\sin t)\left(R(\cos t + i\sin t)\right)^k$$
$$= R(\cos t + i\sin t)R^k(\cos kt + i\sin kt)$$
$$= R^{k+1}\left(\cos kt\cos t - \sin kt\sin t + i(\cos t\sin kt + \sin t\cos kt)\right)$$
$$= r^{k+1}(\cos(k+1)t + i\sin(k+1)t)$$

Therefore, by mathematical induction it is proven that P_n is true for k>1