BBMerlion

Gravitational Field

- **Gravitational field strength** at a point is defined as the gravitational force per unit mass at that point

$$g = \frac{F_g}{m} = \frac{GM}{r^2}$$

g is the free fall acceleration
G = universal gravitational constant = 6.67 x $10^{-11}\ N\ m^2\ kg^{-2}$

- **Newton's law of gravitation**
The (mutual) gravitational force F between two point masses M and m separated by a distance r

$$F = \frac{GMm}{r^2}$$

Example 1:

Assuming that the Earth is a uniform sphere of radius 6.4×10^6 m and mass 6.0×10^{24} kg, find the gravitational field strength g at a point:

(a) on the surface,

$g = GM / r^2 = (6.67 \times 10^{-11})(6.0 \times 10^{24}) / (6.4 \times 10^6)^2 = 9.77 ms^{-2}$

(b) at height 0.50 times the radius of above the Earth's surface.

$g = GM / r^2 = (6.67 \times 10^{-11})(6.0 \times 10^{24}) / ((1.5 \times 6.4 \times 10^6)2 = 4.34 ms^{-2}$

Example 2:

The acceleration due to gravity at the Earth's surface is $9.80ms^{-2}$. Calculate the acceleration due to gravity on a planet which has the same density but twice the radius of Earth.

$g = GM / r^2$

$gP / gE = M_P r_E^2 / M_E r_P^2 = (4/3)\pi r_P^3 r_E^2 \rho_P / (4/3)\pi r_E^3 r_P^2 \rho_E = r_P / r_E = 2$

Hence $g_P = 2 \times 9.81 = 19.6ms^{-2}$

Example 3:

A ship is at rest on the Earth's equator. Assuming the earth to be a perfect sphere of radius R and the acceleration due to gravity at the poles is g_o, express its apparent weight, N, of a body of mass m in terms of m, g_o, R and T (the period of the earth's rotation about its axis, which is one day).

At the North Pole, the gravitational attraction is $F = GM_E m / R^2 = mg^o$

At the equator,
Normal Reaction Force on ship by Earth = Gravitational attraction - centripetal force
$N = mg_o - mR\omega^2 = mg_o - mR\,(2\pi / T)^2$

- **Gravitational potential**: the work done (by an external agent) in bringing a *unit* mass from infinity to a particular point (without changing its kinetic energy)

$$\phi = \frac{W}{m} = -\frac{GM}{r}$$

- Why is it negative?
- the gravitational force on the mass is attractive, the work done by an ext agent in bringing unit mass from infinity to any point in the field will be negative work (as the force exerted by the ext agent is opposite in direction to the displacement to ensure that $\Delta KE = 0$)

- **Gravitational potential energy, U:** is the work done in bringing that mass *m* from infinity to a point in the gravitational field of mass *M*

$$U = m\,\phi = -\frac{GMm}{r}$$

Infinity is used as reference point

- **Total Energy** of a Satellite = Gravitational Potential Energy + Kinetic Energy

$$E_{sat,total} = GPE + Orbital\ KE = -\frac{GMm_{sat}}{r} + {}^{1}/_{2}\left(\frac{GMm_{sat}}{r}\right)$$

$$E_{sat,total} = -\frac{GMm_{sat}}{2r}$$

- **Escape Speed of a Satellite**
- By conservation of energy

$$KE_i + PE_i = KE_f + PE_f$$

$$\,^{1}/_{2}\, m{v_{esc}}^{2} + \left(-\frac{GMm}{r}\right) = 0 + 0$$

Hence, escape velocity of satellite is:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

- For a satellite in circular orbit, "***<u>the centripetal force is provided by the gravitational force</u>***"
  (Must always state what force is providing the centripetal force before following eqn is used!)

- A satellite does not move in the direction of the gravitational force {ie it stays in its circular orbit} because: the gravitational force exerted by the Earth on the satellite is **<u>just sufficient</u>** <u>to cause the centripetal acceleration but not enough to also pull it down towards the Earth</u>

- **Geostationary satellite** is one which is <u>always above a certain point on the Earth</u> (as the Earth rotates about its axis.)

- For a **geostationary** orbit: T = 24 hours, orbital radius (& height) are fixed values from the centre of the Earth, angular velocity, ω, is also a fixed value; rotates from west to east.