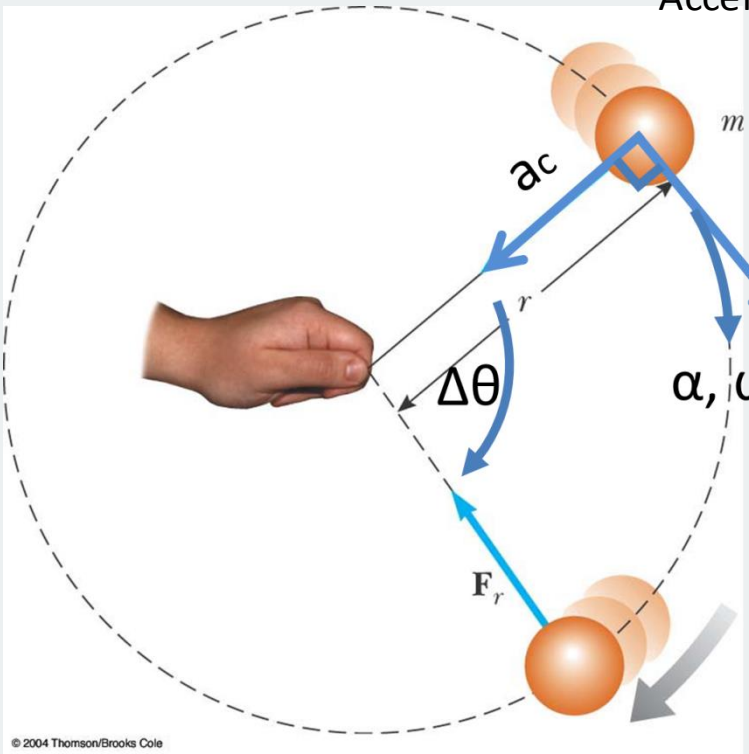
BBMerlion

Motion in Circle

Motion in Circle

- Displacement
  - Angular: $\Delta\theta\ rad$
  - Linear: $\mathbf{s} = \mathbf{r}\,\Delta\boldsymbol{\theta}\ \ \boldsymbol{metre}\ (\boldsymbol{m})$
- Velocity
  - Angular: $\boldsymbol{\omega} = \frac{\Delta\theta}{\Delta t}\ (^{rad}/_{s})$
  - Linear: $\mathbf{v} = \boldsymbol{\omega}\,\mathbf{r}\ (^{m}/_{s})$
- Acceleration
  - Angular: $\boldsymbol{\alpha} = \frac{\Delta\omega}{\Delta t}\ \ (^{rad}/_{s^2})$
  - Linear: $\boldsymbol{a} = \boldsymbol{\alpha}\,\boldsymbol{r}\ \ (^{m}/_{s^2})$
  - Centripetal: $\boldsymbol{a_c} = \frac{v^2}{r} = \boldsymbol{\omega^2 r}\ \ (^{m}/_{s^2})$



***Centrifugal:** centrifugal acceleration has the same value of centripetal. It is the acceleration felt by the object undergoing circular motion. It is a fictitious quantity. **Centripetal acceleration** is the acceleration component that makes an object going a circular motion, and it is directed toward the center of the circular path, along the radius.

Kinematics of uniform circular motion

- Displacement
  - SI unit for θ is radian (rad). θ = angular displacement
  - 1 revolution = 360° = 2π rad
  - Length of arc travelled:

$$s = r\,\theta$$

- Velocity
  - Angular velocity is the rate of change of angular displacement with respect to time :

$$\omega = \frac{\Delta\theta}{\Delta t}$$

  - If T (period, second) or f (frequency, Hz) is known:

$$\omega = \frac{2\pi}{T} = 2\pi f; \quad f = \frac{1}{T}$$

- Linear/tangential velocity ($\vec{v}$) is instantaneous velocity at particular point. The direction of $\vec{v}$ is always tangent to the circular path.

- ω is the same at any point on the rotating object, where v is proportional to r

- *A body moving in a circle at a constant speed changes velocity (since its direction changes). Thus, it always experiences an acceleration, a force and a change in momentum.

- Conversion from rpm (revolution per minute) to radian per second: ____ **rpm x** $\frac{2\pi}{60}$ **=** ____ **rad/s**

Kinematics of uniform circular motion

- Acceleration
  - To calculate the total acceleration, sum up the centripetal acceleration and the linear acceleration by vector addition

$$\vec{a}_{total} = \vec{a}_{centripetal} + \vec{a}_{linear}$$

$$\vec{a}_{centripetal} \perp \vec{a}_{linear}$$

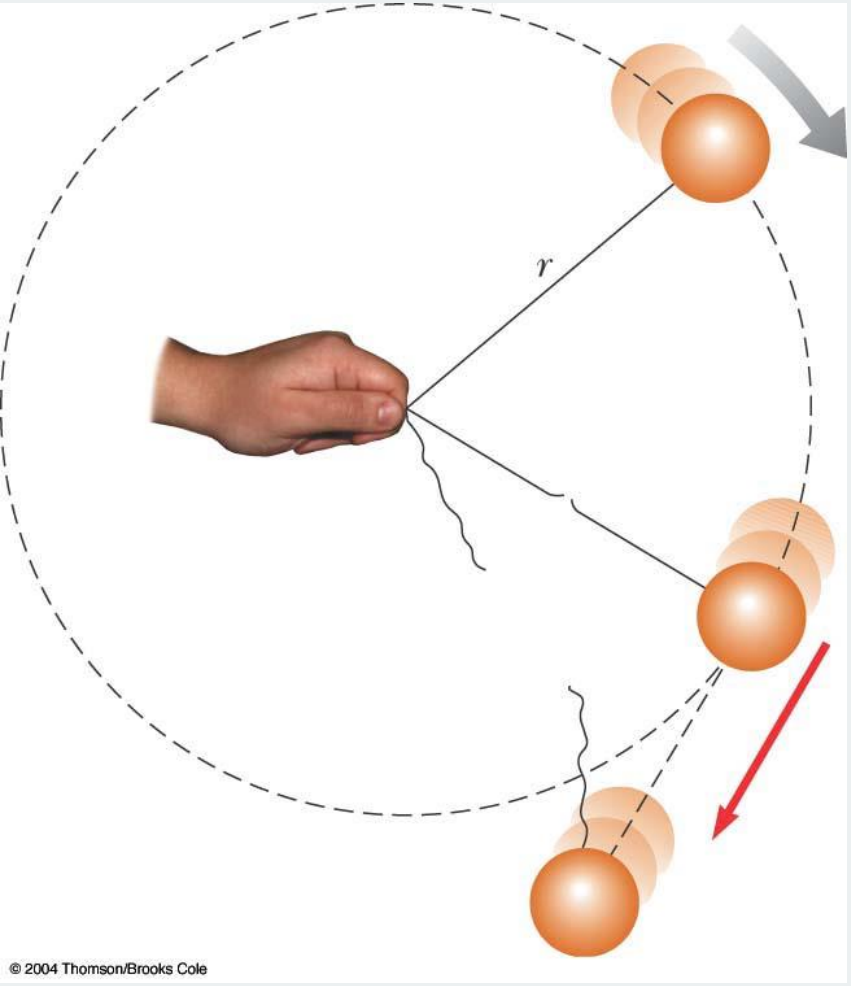
  - α is the same at any point on the rotating object, where *a* is proportional to r

Angular (rotational) vs. linear (translational) kinematic equation

Translational	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2} a t^2$	$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

Centripetal Force and Acceleration

- Centripetal acceleration is caused by centripetal force
- An object moving in uniform circular motion will still experience acceleration, even though $\alpha = 0$ – it is called centripetal acceleration
- The centripetal force is the force that makes the object move in circular motion.



- If the force is gone, then the object will continue to move in a straight line, following the direction of velocity when the centripetal force is gone
- **Centripetal force is a resultant force** of all forces acting in radial direction

- Centripetal force can be calculated as

$$\sum F_c = m\, a_c = m\frac{v^2}{r} = m\, \omega^2\, r$$