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Forces

Ichsan Prasetya

Learning Objectives

- ✓Understand types of force
- ✓Understand the centre of gravity and turning effect
- ✓Understand motion in a circle
- ✓Undestand gravitational field

Forces

Forces: rate of change of momentum

One Newton: is defined as the force needed to accelerate a mass of 1 kg by 1 m s^{-2}.

Mass: is a measure of the amount of matter in a body, & is the property of a body which resists change in motion.

Hooke's law

- Hooke's Law (within limit of proportionality):

$$F = kx$$

F = force/N ; k = force constant/ (N/m); x = length of extension/m

- Elastic potential energy/strain energy = Area under the F-x graph

$$E = \tfrac{1}{2} F x = \tfrac{1}{2} k x^2$$

Types of Force

Force is divided into two categories

Conservative force	**work done in moving a particle between two points does not depend on the path taken (eg. gravitational force and spring force)**
Non-Conservative force	**work** done in **moving** a particle between two points does **depends** on the **path taken** (eg. frictional force)

Types of Force

Gravitational Force	**Attractive force which experienced by mass due to the presence of another mass**
Electric Force	**Attractive or repulsive** force which **experienced by Charge** due to the **presence** of **another Charge**
Upthrust	**Upward** force by the **fluid**, experienced by submerged or floating object due to the **pressure difference** between the **upper** surface and **lower surface** of the object

Types of Force

Frictional Forces	**Occur between two surfaces and always opposes relative motion, the value varies up to a maximum value (static friction)**
Viscous Forces	Occur between the object and **fluid** and always **opposes relative motion**, the value increases as the speed increases

Types of Force

- Archimedes' Principle: Upthrust = weight of the fluid displaced by submerged object.

Fluid Upthrust = Vgh

Vol = Volume/m^3 g = gravitational accelaration/9.81m/s^2, h = depth/m

- Hydrostatic Pressure p = ρgh

ρ = density / (kg/m^3)

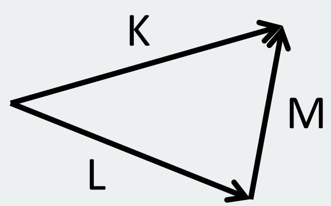
- Centripetal forces = m$\frac{v^2}{r}$ = $m\omega^2$r

m = mass/kg, v = velocity/ (m/s), ω = angular velocity / (rad/s)

Example 1

2 Forces, K and L acted on an object. A force M holds the body in equilibrium. Determine which of the vector diagram below can represent the above statement

A



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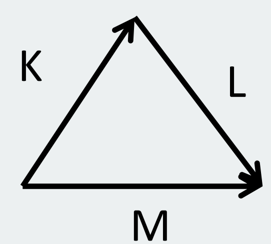

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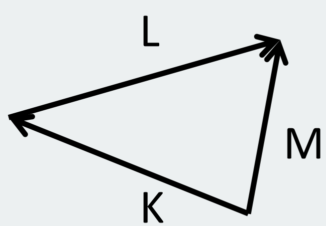

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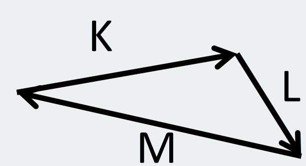


Example 1

Ans: D

To maintain equilibrium, closed triangle (head-to-tail) has to be formed

Turning Effect

- Centre of Gravity of an object is defined as that point through which the entire weight of the object may be considered to act.
- Couple: a pair of forces which tends to produce rotation only.
- Moment of a Force: The product of the force and the perpendicular distance of its line of action to the pivot
- Torque of a Couple**:** The produce of one of the forces of the couple and the perpendicular distance between the lines of action of the forces (**NOT** an action-reaction pair as they act on the same body.)

Example 2

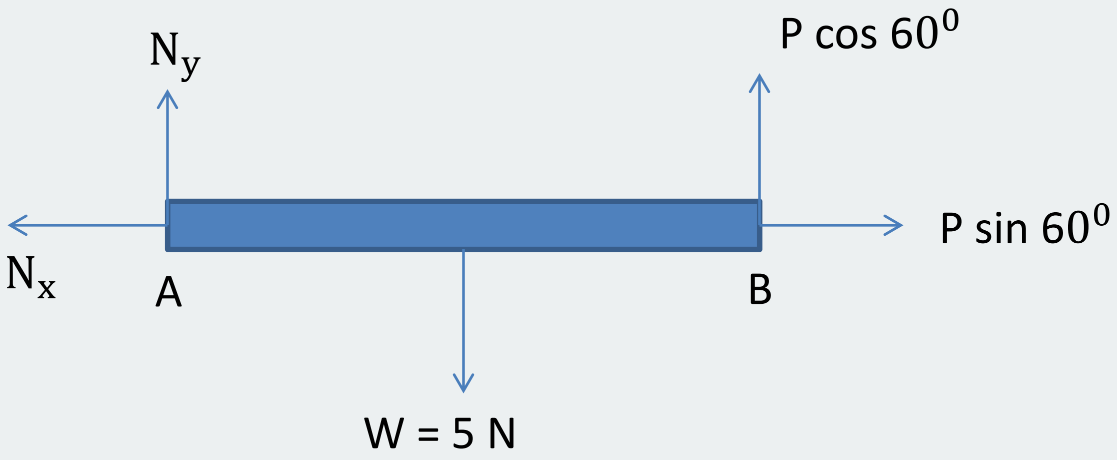
- A rod is freely hinged on point A. The force P is acted 60 degree to the vertical. Calculate P to make the rod equilibrium if the mass of rod is 0.5 kg. (g = 10 ms^{-2})

Ans: 5.00 N

Example 2

Solution:

Free Body diagram



Due to normal contact Force (N) from the hinge, it will be easier to solve the problem by the equation of moment. By taking moment about point A.

W (0.5 L) = P cos 60^0 (L)

5 (0.5 L) = P cos 60^0 (L)

P = 5 N (ans)

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Motion in a Circle

- 180^0 = 2π radian
- ω = θ / t = 2π / T
- Torque (T) = F r
- T = I α, where α = angular accelaration (rad/s^2)

Moment of inertia *I* = $\sum mr^2$

- Rotational momentum = $I_1 \omega_1$

Conservation of angular momentum

$I_1 \omega_1 = I_2 \omega_2$

Motion in a Circle

- v = arc length / time taken = $r\theta / t = r\omega$
- Centripetal acceleration ; $a = r\omega^2 = v^2 / r$
- centripetal force F = mrω2, F = mv2/r

Example 3

A disc is rotating on a flat frictionless table around the verticle axis. The point A is located on the disc twice as far as point B from the centre of the disc. At a given instant state the ration of angular and linear velocity of A to that of B?

Ans: 1, 2

Solution:

Since Point A and B is located on the same disc the angular velocity is the same, so $\frac{\omega_A}{\omega_B} = 1$, because

$$v_A/v_B = \frac{\omega_A r_A}{\omega_B r_B} = 2$$

Gravitational Field

- Gravitational force (F)= $\frac{Gm_1m_2}{r^2}$
- Gravitational field strength (g)= $\frac{GM}{r^2}$, gravitational force per unit mass at that point
- Gravitational potential ϕ= - $\frac{GM}{r^2}$
- Gravitational potential energy, U = m φ = - GMm / r
- G = universal gravitational constant = 6.67 x $10^{-11}\ N\ m^2\ kg^{-2}$

Gravitational Field

Espeed of satellite is the velocity needed for a satellite to escape the earth and carry on with constant velocity

Initial KE + initial GPE = final KE + final GPE

$0.5mv^2 + (-GMm / r) = 0 + 0$

Hence $v = \sqrt{(2GM / R)}$

Example

A planet has gravitational field strength g, what is the new gravitational field strength of the planet if the planet has twice the initial radius? State the assumption you made and answer in terms of g.

Ans: 2 g

Example

Solution: twice initial radius **doesn't necessarily** make the mass of the planet become twice as big as the initial mass. Assume the mass density of the planet doesn't change.

$$g_2/g_1 = \frac{GM_2}{r_2{}^2} / \frac{GM_1}{r_1{}^2}$$

$$M_1 = \rho V_2 = \rho\, 4/3\pi(r_1{}^3)$$

$$g_2/g_1 = \frac{M_2}{r_2{}^2} / \frac{M_1}{r_1{}^2}$$

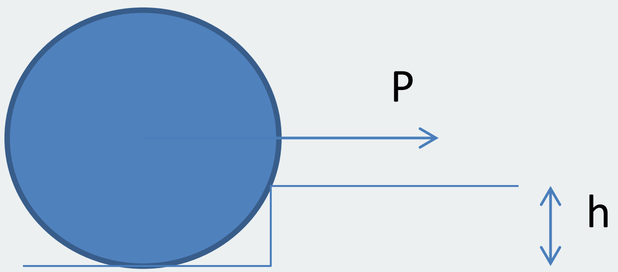
$$M_2 = \rho V_2 = \rho\, 4/3\pi(2r_1)^3$$

$$g_2/g_1 = \frac{8M_1}{4r_1{}^2} / \frac{M_1}{r_1{}^2} = 2$$

Hence $g_2 = 2g_1$

Example

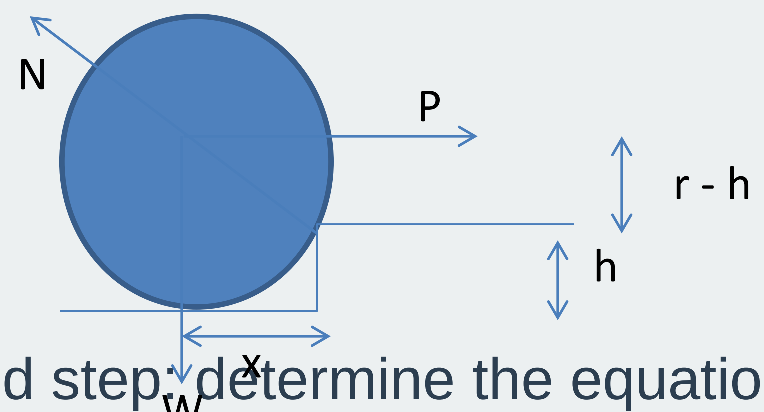
Determine the minimum magnitude of P to lift up the disc in terms of acceleration of free fall (g), mass (m) and radius (r) of the disc. And determine magnitude of reaction force by the curb on the disc



Example

Solution:

1st step: draw free body diagram



2nd step: determine the equation of moment. Consider the moment around the edge of the curb

$$x = \sqrt{r^2 - (r - h)^2} = \sqrt{2rh - h^2}$$

Example

In the situation when the disc started to raise, no contact force exerted on the disc by the floor, so the only force acting is Weight (W), Normal force from crub to the disc (N), and force P

clockwise moment = anti-clockwise moment

$$P\ (r-h) = mg\ \sqrt{2rh - h^2}$$

$$P = \frac{mg\ \sqrt{2rh - h^2}}{(r-h)}\ \textbf{(ans)}$$

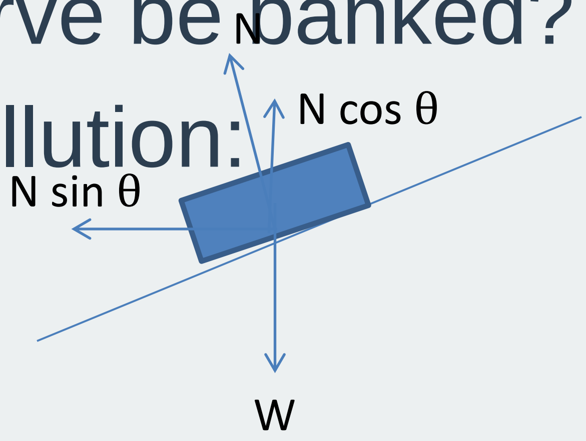
In equilibrium condition the 3 forces make a close triangle, so:

$$N = \sqrt{W^2 + P^2} = \sqrt{(mg)^2 + \frac{(mg)^2(2rh - h^2)}{(r-h)^2}} = \frac{mgr}{r-h}\ \text{(ans)}$$

Example

A car travelling a frictionless banked curve (radius 60 m), with velocity = 70 km/h, at what angle the to the horizontal should the curve be banked?

Sollution:



70 km/h = 7000 m/3600 s = 19.44 m/s

$\text{N} \cos\theta = \text{mg}$

$\text{N} \sin\theta = \dfrac{\text{mv}^2}{\text{r}}$

$\tan\theta = \dfrac{\text{v}^2}{\text{rg}};\ \theta = \tan^{-1}\left(\dfrac{19.44^2}{60 \text{ x } 9.81}\right) = 32.7^0$

Example

Newton was able to calculate the ratio of the mass of the sun to the mass of any planet without knowing the numerical value of universal gravitational constant G, provided the planet has a satellite.

a) Show for a circular orbit

$$\frac{M_s}{M_p} = \left(\frac{R_p}{R_m}\right)^3 \left(\frac{T_m}{T_p}\right)^2$$

M_s = mass of sun; Mp = mass of the planet; R_p = distance of planet from the sun; R_m = distance of moon from planet; T_m = period of mood around planet; T_p = period of planet around the sun

b) If $R_p = 1.5 \times 10^8$ km, $R_m = 3.85 \times 10^5$ km, $T_m = 27.3$ days, and $T_p = 365.2$ days, calculate $\frac{M_s}{M_p}$.

Example

Solution

a) Centripetal force = Gravitational force

$$mr\omega^2 = \frac{GMm}{r^2} \qquad mr\left(\frac{2\pi}{T}\right)^2 = \frac{GMm}{r^2}$$

Hence, $M = \frac{4\pi^2 r^3}{GT^2}$; $M_s = \frac{4\pi^2 {r_p}^3}{G{T_p}^2}$; $M_p = \frac{4\pi^2 {r_m}^3}{G{T_m}^2}$

$$\frac{M_s}{M_p} = \frac{4\pi^2 {r_p}^3}{G{T_p}^2} / \frac{4\pi^2 {r_m}^3}{G{T_m}^2} = \left(\frac{R_p}{R_m}\right)^3 \left(\frac{T_m}{T_p}\right)^2 \text{ (shown)}$$

b) $\frac{M_s}{M_p} = \left(\frac{1.5 \times 10^8}{3.85 \times 10^5}\right)^3 \left(\frac{27.3}{365.2}\right)^2 = 3.30 \times 10^5$ (ans)