BBMerlion

Particle Mechanics

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KINEMATICS AND DYNAMICS

Objective

✓Understand the kinematics of uniformly accelerated linear motion

✓Understand the dynamics of particle moving under forces

✓Analyze the cases of motion involving pulley, strings, and springs

✓Understand the concept of uniform circular motion

Definitions

Kinematics:

a study of motion without the consideration of its causes

Involves:

position, velocity, acceleration

Dynamics:

a study of forces (and torques) and their effects on motion

Involves:

kinematics, forces, interaction, time-variation

Kinematics

Basic Equations

Velocity

Instantaneous velocity

$$v = \frac{dx}{dt}$$

Average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\sum v_i t_i}{\sum t_i}$$

Acceleration

Instantaneous acceleration

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

Average acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

Position

$$x = \int vdt = \int \frac{vdv}{a}$$

Kinematics

Uniform linear motion & linearly accelerated motion

For a constant velocity:

$$\boxed{s = vt}$$

For a constant acceleration:

$$v = u + at$$

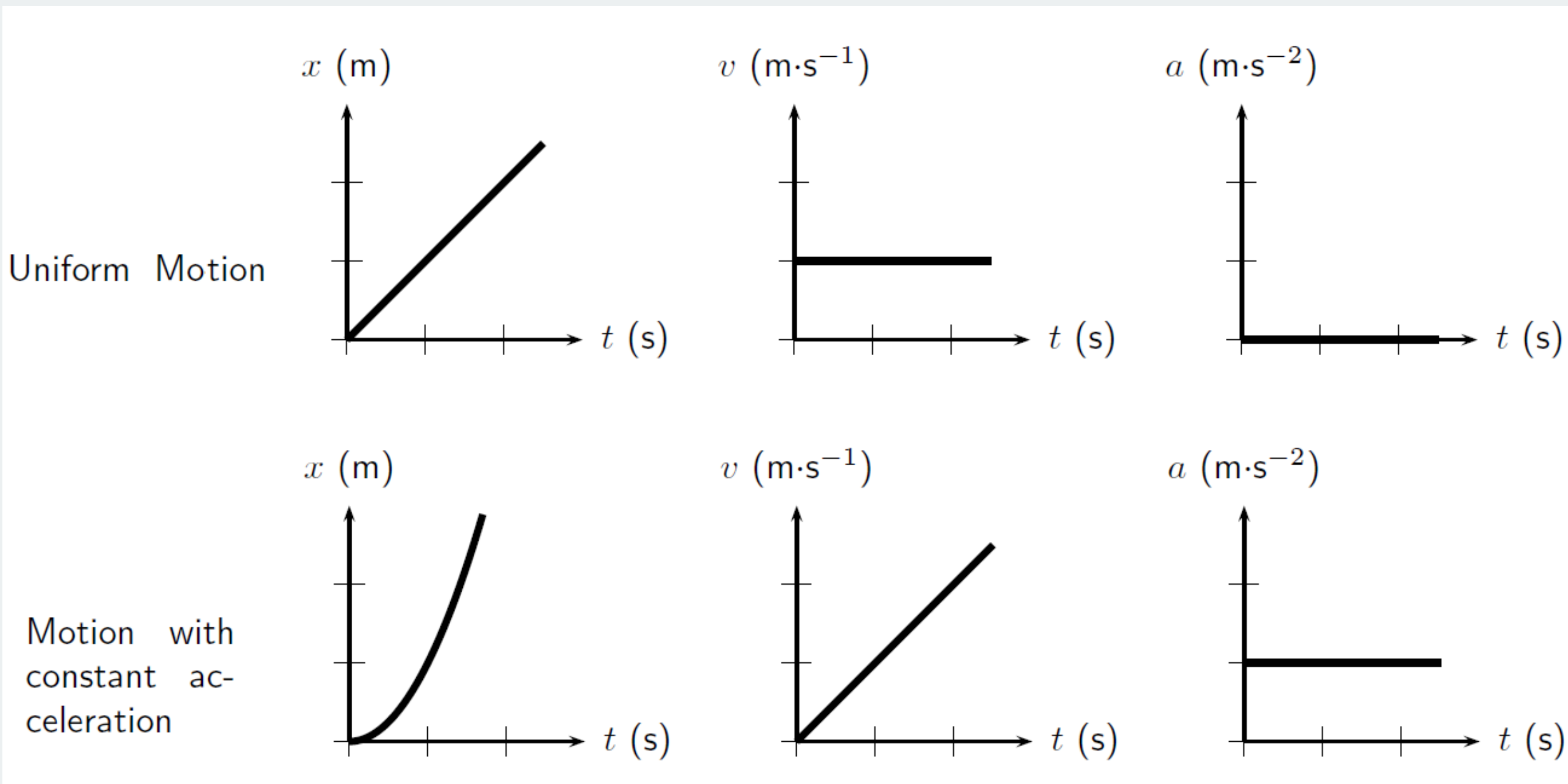
$$s = \int v \, dt = ut + \frac{1}{2}at^2$$

$$s = t\left(\frac{2u + at}{2}\right) = t\left(\frac{u + v}{2}\right)$$

$$s = \int \frac{vdv}{a} = \frac{v^2 - u^2}{2a} \quad => \quad v^2 = u^2 + 2as$$

Kinematics

Graphs



(source: http://askmichellephysics.blogspot.com/)

Example 1a

Graph of position versus time

Problem

A particle moves along the x axis. Its position varies with time according to the expression $x(t) = (-4t + 2t^2)\,m$

Sketch the (x-t) graph of the above equation, and

a) Determine the displacement of the particle in the time intervals (t = 0 to 1s) and (t = 1s to 3s).

b) Calculate the average velocity during these two time intervals.

c) Find the instantaneous velocity of the particle at *t = 2.5 s.*

(source: NTU FE1011 Lecture Notes, 2011)

Example 1a

Graph of position versus time

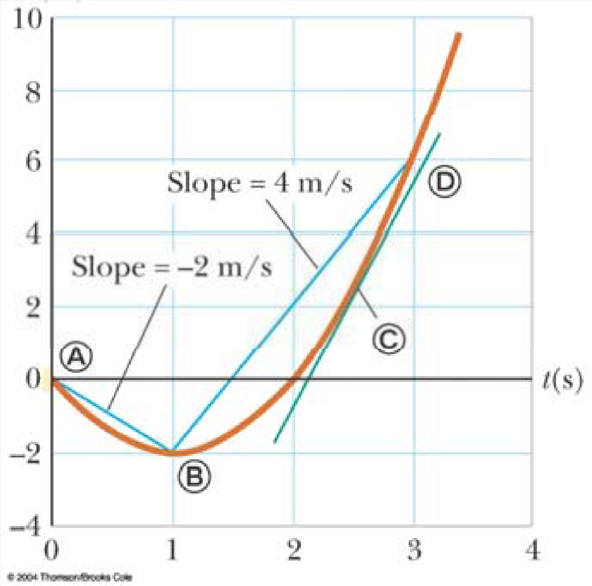
Solution

a) As seen from the graph, the displacement from t = 0 to 1s is shown by the line A-B

$x(1) - x(0) =$ **-2 m**

And the displacement from t = 1s to 3s is shown by the line B-D

$x(3) - x(1) =$ **8 m**



$x(t) = (-4t + 2t^2)\, m$

Example 1a

Graph of position versus time

Solution

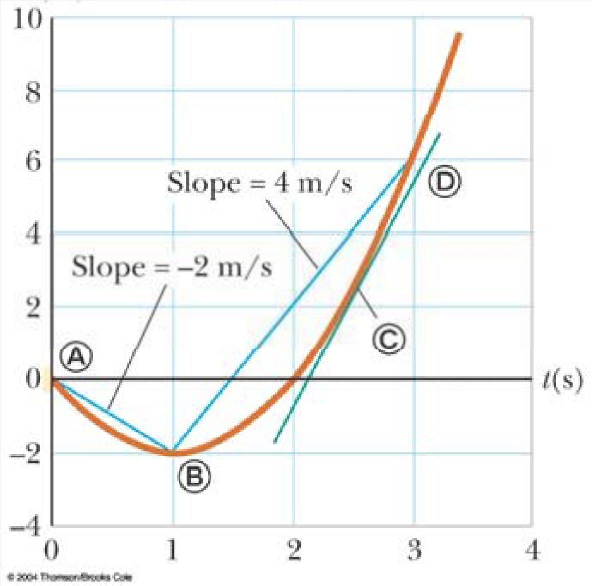
b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{\sum v_i t_i}{\sum t_i}$

The average velocity from t = 0 to 1s is shown by the slope A-B

$$\bar{v} = \frac{-2-0}{1-0} = -2\, m/s$$

And the average velocity from t = 1s to 3s is shown by the slope B-D

$$\bar{v} = \frac{6-(-2)}{3-1} = 4\, m/s$$



$\underline{x(t) = (-4t + 2t^2)\, m}$

Example 1a

Graph of position versus time

Solution

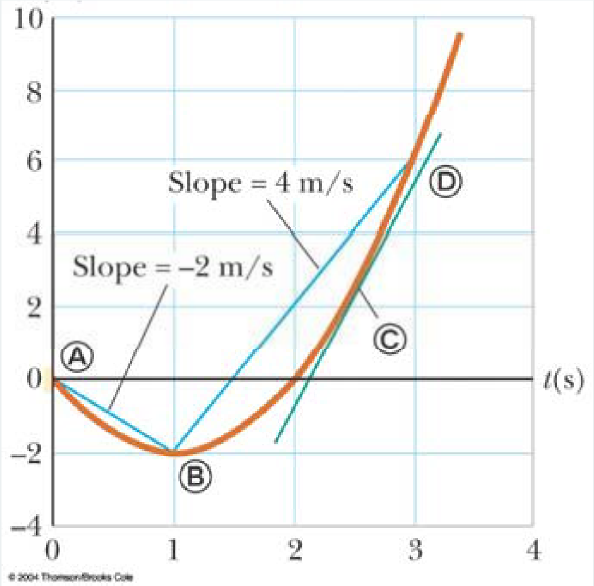
c) $v = \frac{dx}{dt}$

v(t) = -4 + 4t

The instantaneous velocity at t = 2.5 s, as shown by the slope of tangential line at C, is

$v(\frac{5}{2})$= -4 + 4(2.5)

= **6 m/s**



$x(t) = (-4t + 2t^2)\ m$

Example 1b

Graph of velocity versus time

Problem

The velocity of a particle moving along the x axis varies in time according to the expression $v_x(t) = (40 - 5t^2)\ {}^{m}/_{s}$

Sketch the (v-t) graph of the above equation, and

a) Find the average acceleration in the time interval t = 0 to 2s.

b) Determine the acceleration at t = 2s.

(source: NTU FE1011 Lecture Notes, 2011)

Example 1b

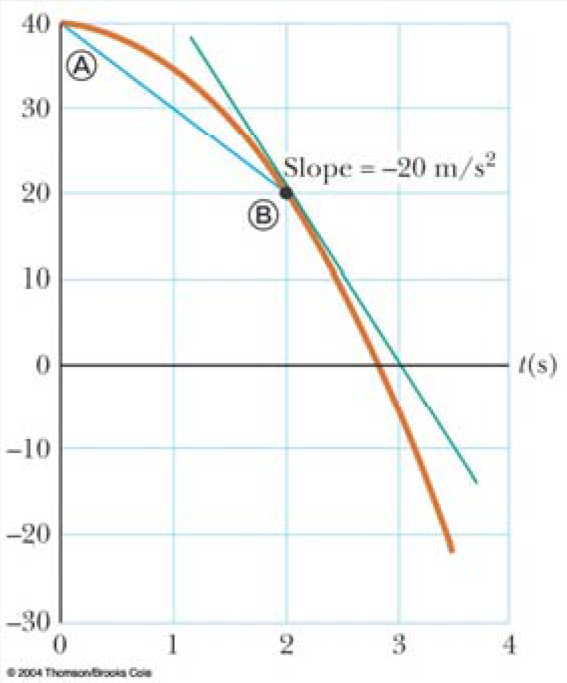
Graph of velocity versus time

Solution

a) $\bar{a} = \dfrac{\Delta v}{\Delta t}$

As seen from the graph, the average acceleration from t = 0 to 2s is shown by the slope A-B

$$\bar{a} = \frac{20 - 40}{2 - 0} = -10\ {}^{m}/_{s^2}$$



$\underline{v_x(t) = (40 - 5t^2)\ {}^{m}/_{s}}$

Example 1b

Graph of velocity versus time

Solution

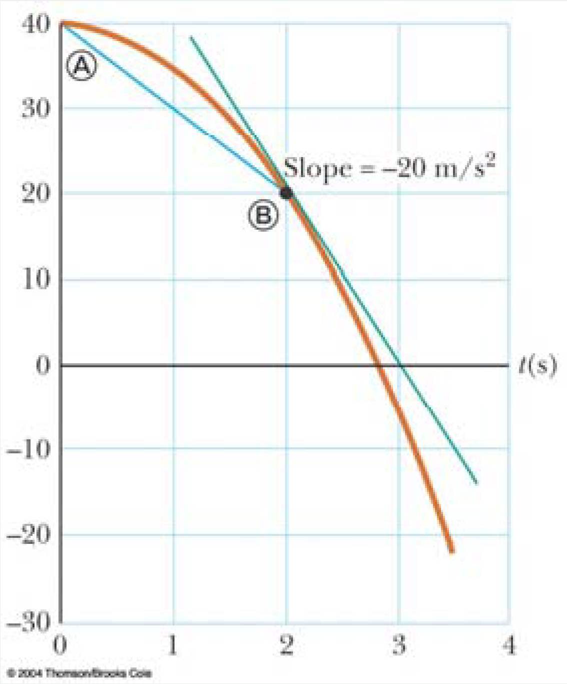
b) $a = \frac{dv}{dt}$

a(t) = -10t

The instantaneous acceleration at t = 2s, as shown by the slope of tangential line at B, is

a(2) = -10(2)

= **-20 m/s²**



$v_x(t) = (40 - 5t^2)\ {}^{m}/_{s}$

Dynamics

Newton's Law of Motion

Newton's 1st Law

$$\sum F = 0$$

*An object at rest will **remain** at rest, an object in motion will **remain** in motion with constant speed in straight line*

Newton's 2nd Law

$$\sum F = m\,a$$

***Resultant** force is equal to **mass x acceleration** with the **same direction** as the acceleration*

Newton's 3rd Law

$$F_{react} = -F_{act}$$

*For a given force there is a **reaction** with the **same magnitude** & **opp. direction** on **two bodies***

Dynamics

Vectors

Unit Vector:

a dimensionless vector with magnitude of 1, to specify a direction without any physical significance.

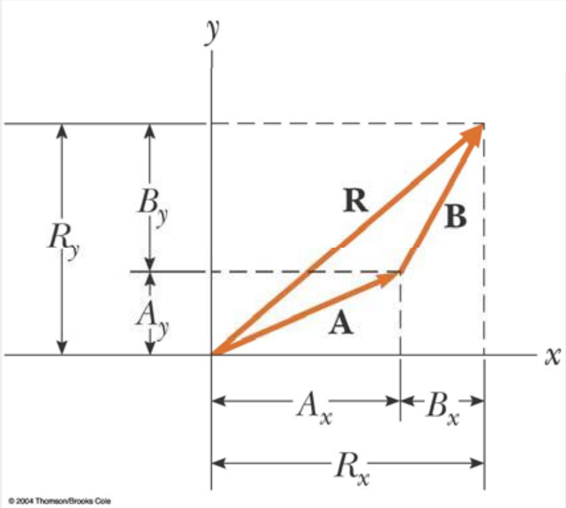
Symbols: $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}$

R = **A** + **B**

$= \left(A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}\right) + \left(B_x\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}}\right)$

$= (A_x + B_x)\hat{\boldsymbol{\imath}} + (A_y + B_y)\hat{\boldsymbol{\jmath}}$

$= R_x\hat{\boldsymbol{\imath}} + R_y\hat{\boldsymbol{\jmath}}$



(source: NTU FE1011 Lecture Notes, 2011)

Dynamics

Friction

Friction Force:

a resistance due to interaction of an object moving on a surface or through a viscous medium.

Friction force is proportional to the normal force.

$\boxed{f_s \le \mu_s n}$ for static friction

$\boxed{f_k = \mu_k n}$ for kinetic friction

Example 2

Forces

Problem

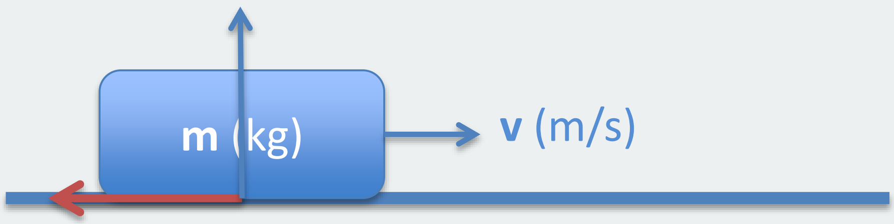
A particle of mass ***m*** kg is travelling in a horizontal straight line with velocity ***u*** m/s. It is brought to rest by means of a resisting force of magnitude ***km(2u-v)***, where ***v*** is the velocity of the particle at any instant and ***k*** is a positive constant. Find the distance travelled by the particle while ***v*** decreases from ***u*** to zero.

(source: NUS A-Level Sample)

Example 2

Forces

Solution



Linear Motion Apps

Pulley System

Two Connected Particles

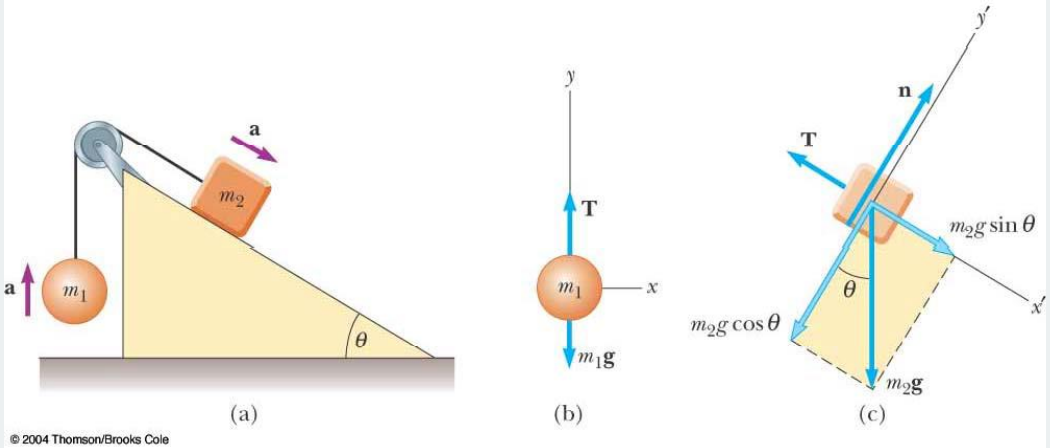
There are forces acting on the objects:

- Tension (one string → same for both object)
- Gravitational force

Each object has the same acceleration since they are connected.

Apply Newton's Law to solve for the unknown(s).

(source: NTU FE1011 Lecture Notes, 2011)



Linear Motion Apps

Elasticity

Elasticity

So far it is assumed that objects remain rigid when external forces are applied. Actually, objects are *deformable*.

Stress: proportional to the force causing deformation per unit area

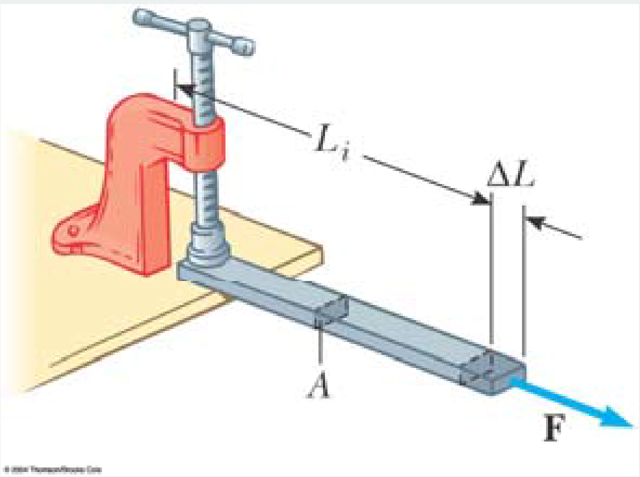
Strain: is a measure of the deformation, as a result of a stress

Elastic Modulus = Stress / Strain

Young's Elastic Modulus

measures change of ***length***

$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{F/A}{\Delta L/L_i}$$



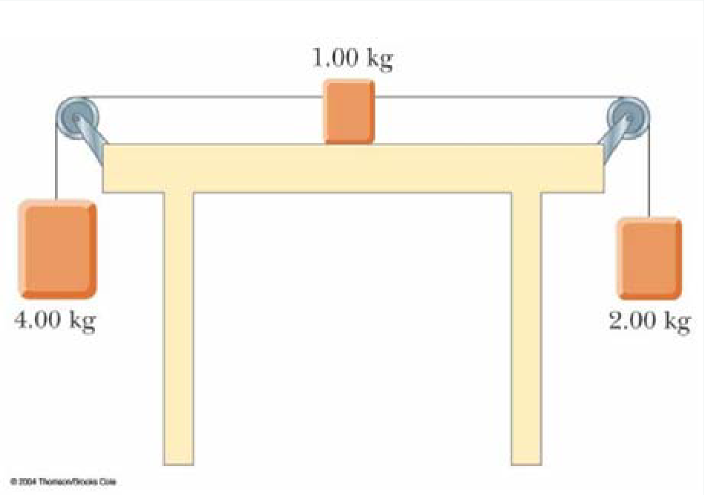
Example 3a

Fixed Smooth Pulley

Problem

Three objects are connected on the table as shown in the following figure. The objects have masses 4.00 kg, 1.00 kg and 2.00 kg, and the pulleys are frictionless. The coefficient of kinetic friction between the 1.00-kg block and the table is 0.350.

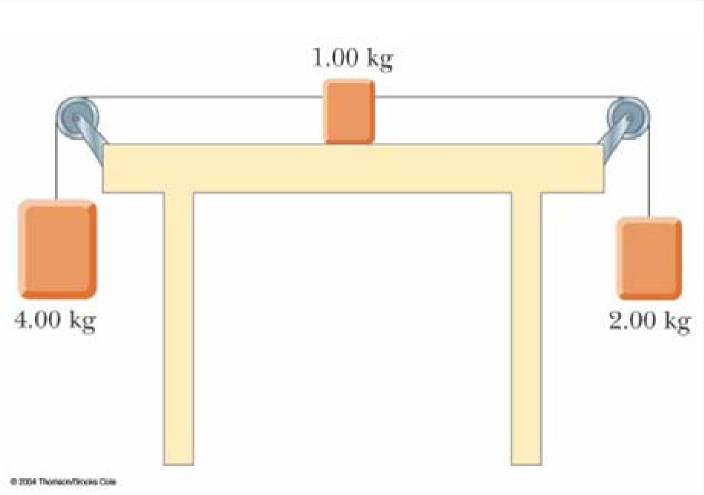
(source: NTU FE1011 Tutorial, 2011)



Example 3a

Fixed Smooth Pulley

Solution



Example 3b

Elastic String

Problem

A light elastic string has a natural length 1m. One end of the string is attached to the fixed point O and a particle P of mass 4kg is suspended from the other end of the string.

a) When hanging in equilibrium, P is 1.2m below O. Find the modulus of elasticity of the string.

b) When P is hanging in equilibrium, it is hit by a particle Q (2kg) which is travelling vertically upwards. Immediately after the impact, P moves vertically with a velocity u m/s. When the string is just taut, P is still moving vertically upwards with a velocity of $\sqrt{10}$ m/s. Find the value of u.

(source: NUS A-Level Sample)

Example 3b

Elastic String

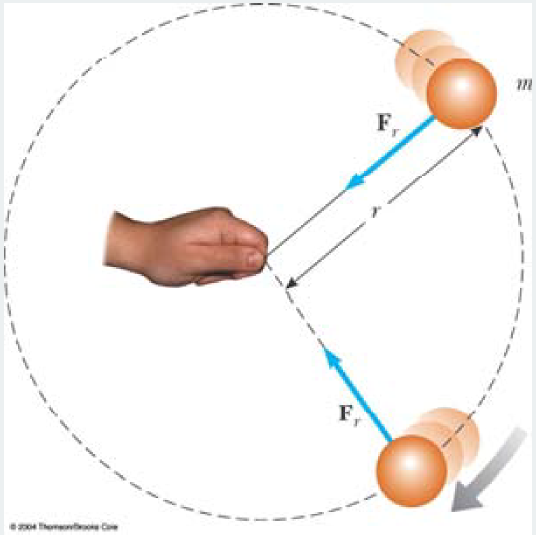
Solution

Circular Motion

Basic Concept

Uniform Circular Motion:

a movement of an object along a circular path, caused by a centripetal force $\boldsymbol{F_r}$, which is associated with the centripetal acceleration $\boldsymbol{a_c}$.



$\boldsymbol{F_r}$ is **not** a **new** force, but instead it is a **new role** for a force.

Applying Newton's Second Law along the radial direction,

$$\sum F = \boldsymbol{F}_r = m\boldsymbol{a}_c = \frac{mv^2}{r}$$

Example 4

Circular Motion

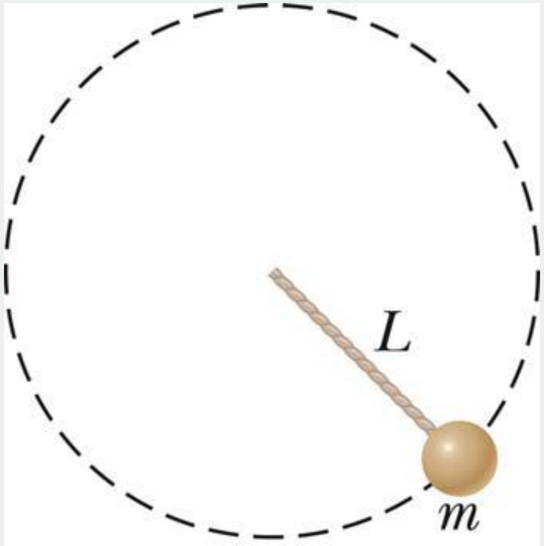
Problem

A ball of mass m = 0.275 kg swings in a vertical circular path on a string L = 0.850 m long as in the following figure.

a) Identify the forces acting on the ball at any point on the path.

b) Draw force diagrams for the ball when it is at the bottom of the circle and when it is at the top.

c) If its speed is 5.20 m/s at the top of the circle, what is the tension in the string there?

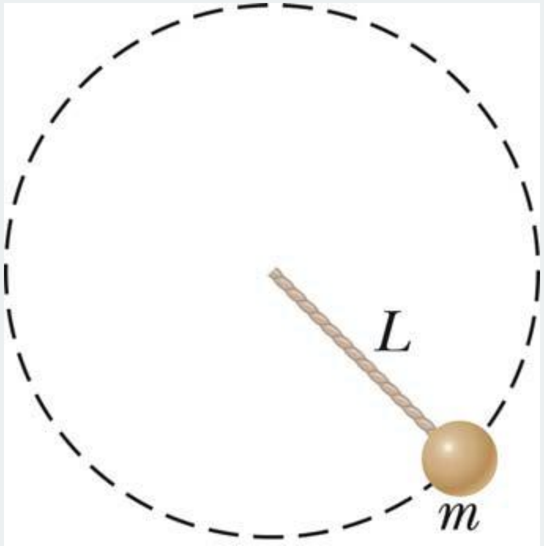


(source: NTU FE1011 Tutorial, 2011)

Example 4

Circular Motion

Solution



ENERGY AND MOMENTUM

Objective

✓Understand the kinetic & potential energy, work, power, and its conservation

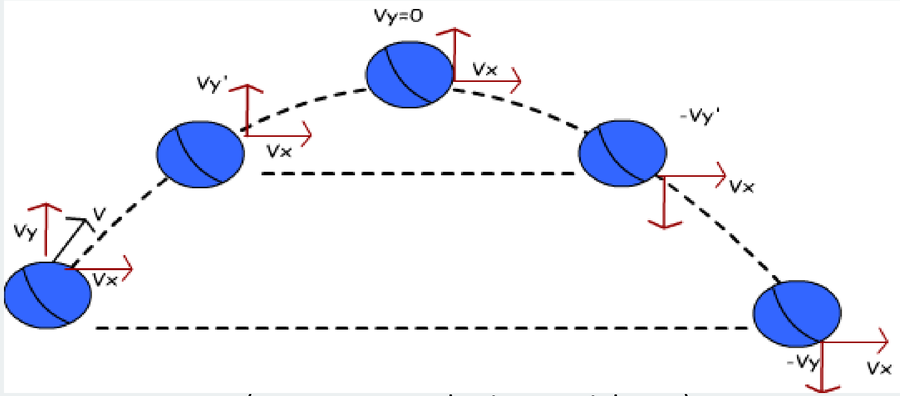
✓Understand the concept of momentum , impulse, and its conservation

✓Analyze the cases of projectile motion

Energy, Work, Power

Motivation

Every physical process that occurs in the universe involves energy, as one of the most important topic in science and engineering.



(source: www.physicstutorials.org)

In practice, some problems might be difficult to solve merely using Newton's Law. These can be made easier with the energy approach.

Energy, Work, Power

Definitions

Energy:

an ability to do Work; a transferable property of objects

Work: $W = \int F.dr$

the presence of a force ***F*** that acts on a body and causes a displacement $\boldsymbol{\Delta r}$ in the direction of the force.

Power: $P = dW/dt$

a rate of doing work; an amount of energy consumed per unit time

Energy

Mechanical Energy

Kinetic Energy:

energy possessed by an object due to its motion.

$$E_k = \frac{1}{2}mv^2$$

Potential Energy:

energy stored in a system due to its position or internal structure.

$$E_g = mgh$$

Gravitational PE

$$E_s = \frac{1}{2}kx^2$$

Elastic PE

Energy

Conservation of ME

<u>Law of Conservation of Energy:</u>

states that total amount of energy in an isolated system is conserved over time.

For a mechanical energy, where $E_m = E_k + E_g + E_s$

$$\boxed{E_{k(final)} + E_{g(final)} + E_{s(final)} = E_{k(init)} + E_{g(init)} + E_{s(init)}}$$

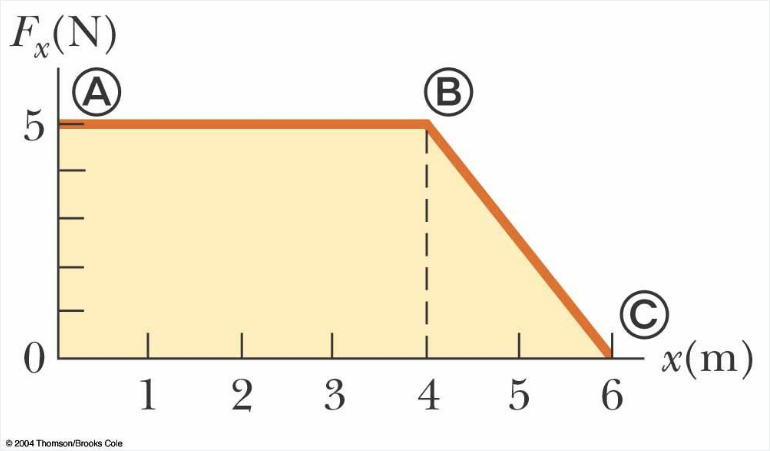
Conservation of Mechanical Energy

Example 5a

Work Done

<u>Problem</u>

A force acting on a particle varies with distance ***x***, as shown below.



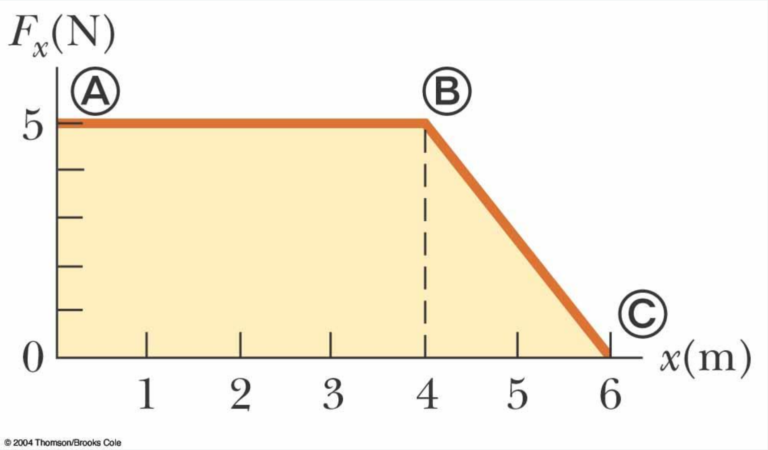
Calculate the work done by the force as the particle moves from ***x*** = 0 to 6.0m.

(source: NTU FE1011 Lecture Notes, 2011)

Example 5a

Work Done

Solution



Example 5b

Power

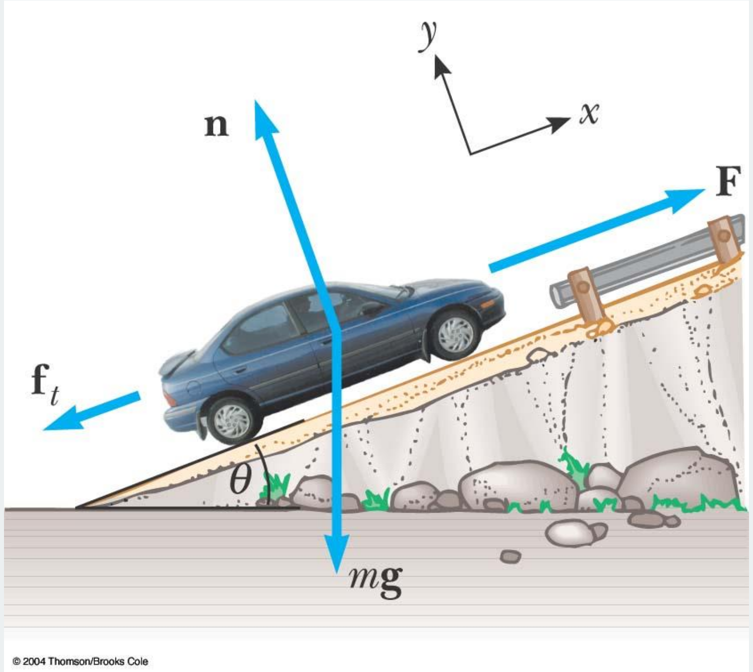
Problem

Consider a car of mass ***m*** that is accelerating up a hill. The total resistive force is

$$f_t = (218 + 0.70v^2)\ N$$

where ***v*** is speed in m/s.

Determine the power that the engine must deliver to the wheels as a function of speed.

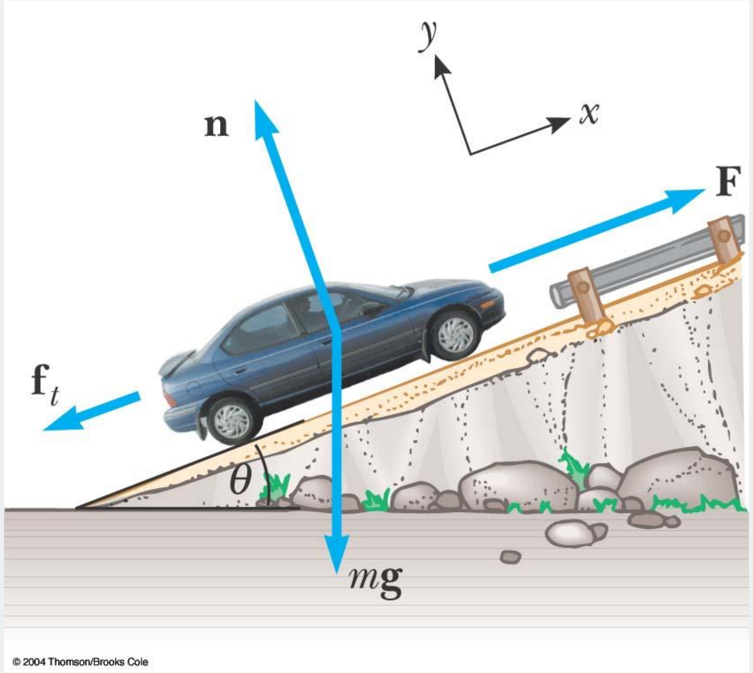


(source: NTU FE1011 Lecture Notes, 2011)

Example 5b

Power

Solution



Example 5c

Conservation of ME

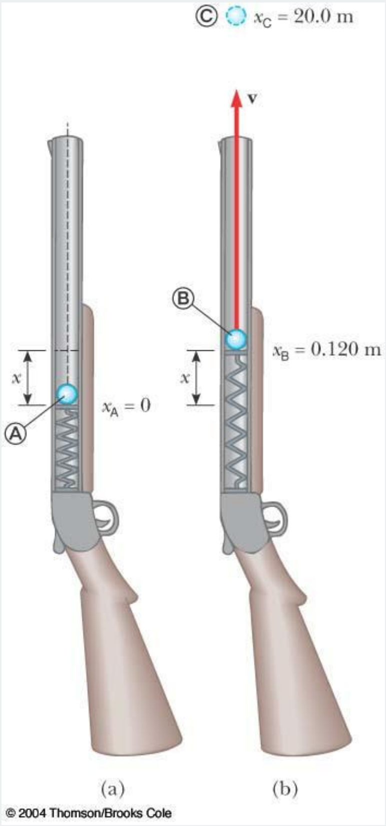
Problem

The launching mechanism of a toy gun consists of a spring with unknown constant ***k***. When the spring is compressed 0.120m **(A)**, the gun is able to launch a 35.0-g projectile to **(C)**, a maximum height of 20.0m above its position before firing.

a) Neglecting all resistive forces, find ***k***.

b) Find the speed of projectile at **(B)**.

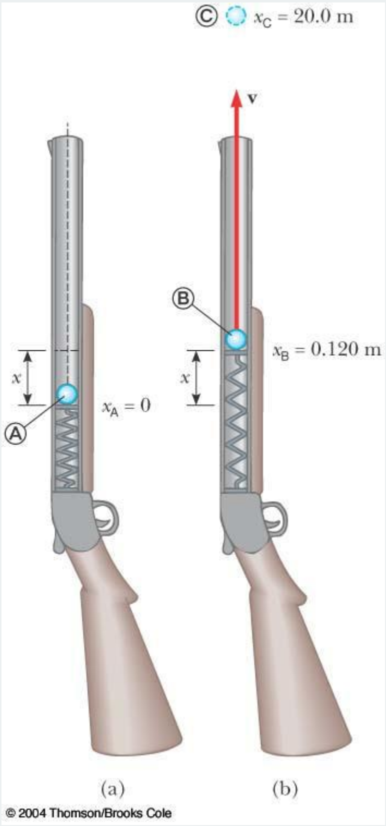
(source: NTU FE1011 Lecture Notes, 2011)



Example 5c

Conservation of ME

Solution



Momentum & Impulse

Concepts

Linear Momentum:

simply a product of the mass $\boldsymbol{m}$ and the velocity vector $\vec{\boldsymbol{v}}$.

$$\boxed{\vec{p} = m\vec{v}}$$

Impulse:

a change in linear momentum produced over an interval.

As from $F = ma = m\dfrac{dv}{dt}$,

$$\boxed{I = \Delta p = m\Delta v = \int m dv = \int F dt = F\Delta t}$$

Momentum

Conservation

<u>Conservation of Linear Momentum:</u>

whenever two or more particles in an <u>isolated system</u> (no resultant external force) interact, the total momentum remains constant.

$$\sum F = \frac{dp}{dt} = 0 \longrightarrow p = \text{constant}$$

$$\boxed{\sum m_{final}\,\vec{v}_{final} = \sum m_{init}\,\vec{v}_{init}}$$

Example 6a

Impulse

Problem

In a crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities of the car are

m/s and

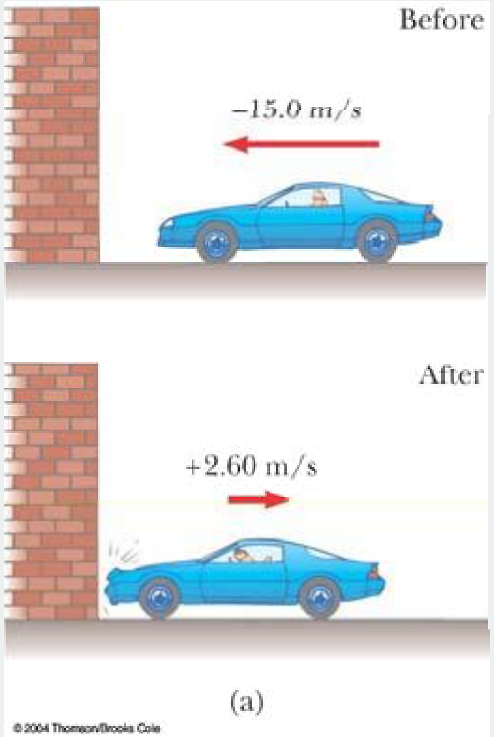
m/s, respectively.

If the collision lasts for 0.150 s, find

a) The impulse caused by the collision;

b) The average force exerted on the car.

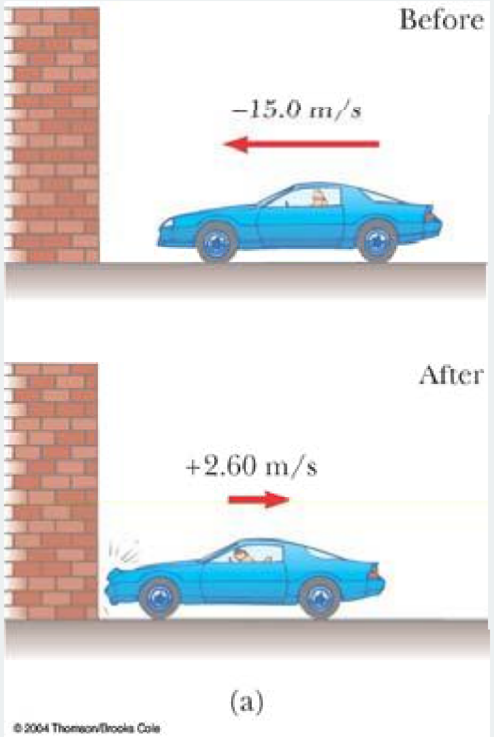
(source: NTU FE1011 Lecture Notes, 2011)



Example 6a

Impulse

Solution



Example 6b

Conservation of Momentum

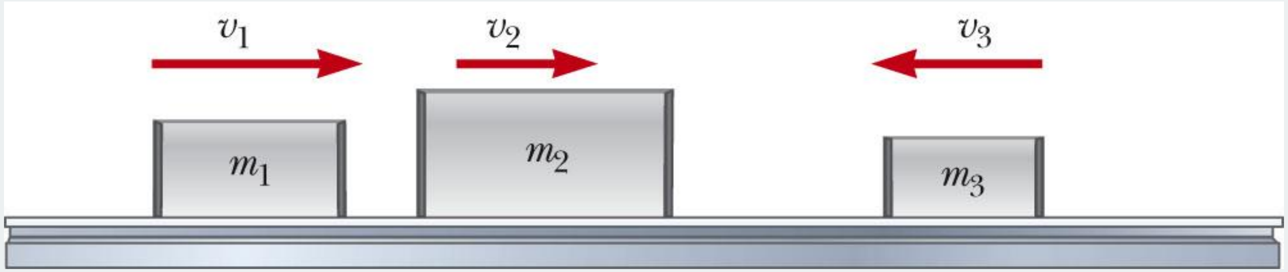
Problem

Three carts move on a frictionless, horizontal track with speeds of v1 = 5.00 m/s (to the right),

v2 = 3.00 m/s (to the right), and

v3 = 4.00 m/s (to the left).

Velcro couplers make the carts stick together after colliding. Find the final velocity of the train of three carts.



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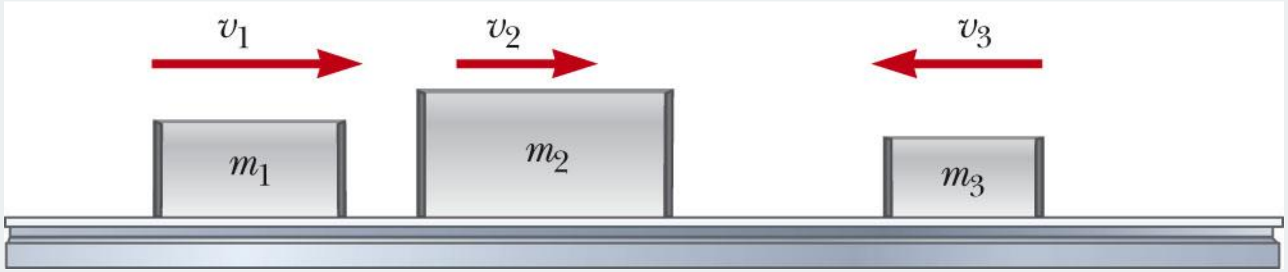
Example 6b

Conservation of Momentum

Solution



Projectile Motion

Components

Projectile Motion:

a movement of an object in both x and y directions simultaneously.

Acceleration Components

$$a_x = 0$$
$$a_y = -g$$

Initial Velocity Components

$$v_{xi} = v_i \cos\theta$$
$$v_{yi} = v_i \sin\theta$$

Displacements

$$x_f = v_{xi}t = (v_i \cos\theta)t$$
$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = (v_i \sin\theta)t - \frac{1}{2}g t^2$$

Projectile Motion

Symmetric Motion

For Symmetric Motion,

The maximum height of the projectile

$$y_{max} = \frac{{v_i}^2 sin^2\theta_i}{2g}$$

The horizontal range of the projectile

$$x_{max} = \frac{{v_i}^2 \sin 2\theta_i}{g}$$

Example 7

Projectile Motion

Problem

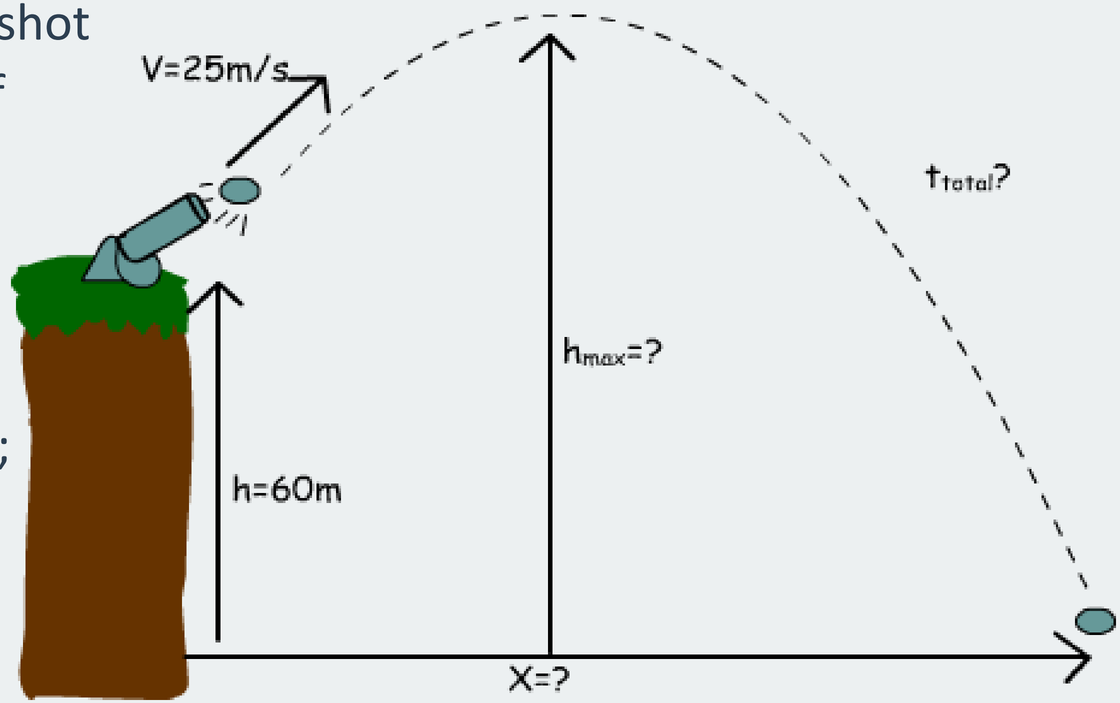
A cannonball is shot with an angle of 53º with the horizontal.

Find:

a) Max height;

b) Horizontal distance;

c) Total time.

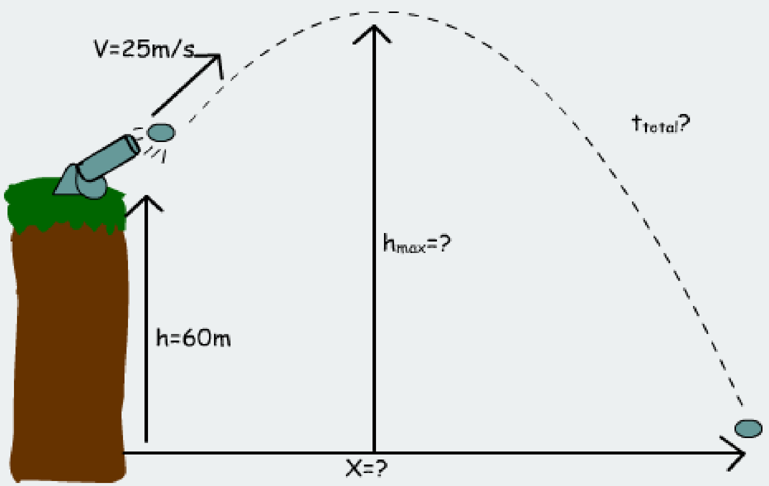


(source: www.physicstutorials.org)

Example 7a

Projectile Motion with Kinematic Equations

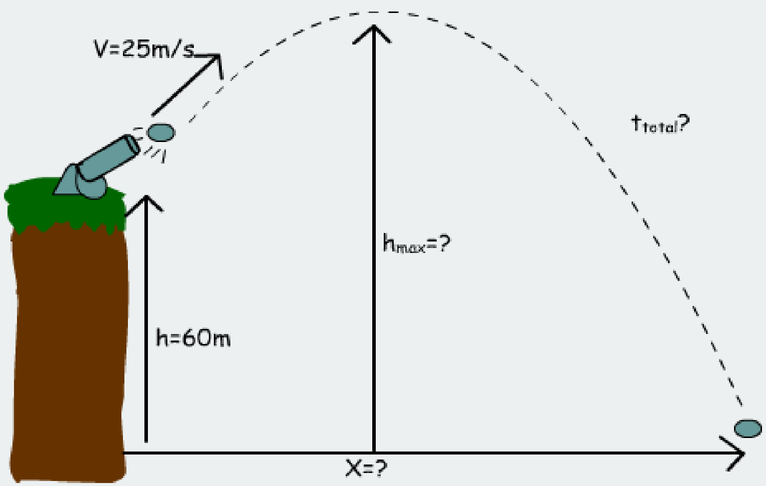
Solution



Example 7b

Projectile Motion with Energy Approach

Solution



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